Advances in Engineering & Scientific Research

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Research Article

TRANSVERSE AND LONGITUDINAL MATRIX CRACKING EVOLUTION IN COMPOSITE LAMINATES: A DAMAGE CRITERION

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Received November 13, 2015; Accepted November 30, 2015; Published December 14, 2015;

Cite This Article: REBIERE, J.(2015). Transverse and longitudinal matrix cracking evolution in composite laminates: a damage criterion. Advances in Engineering & Scientific Research, 1(1).1-8

ABSTRACT

After numerous numerical simulation on the distribution of the stress field in damage composite cross ply laminates we have elaborate an energy criterion. This energy criterion is based on the computation of some partial parts of the strain energy release rate associated with each damage types and for the three loading mode (mode I(opening mode), mode II(sliding mode) and mode III (tearing mode)). In the related criterion, linear fracture based approach, several hypothesis are used to simplify the damage criterion. The main objective with this approach is to estimate the initiation of transverse and longitudinal cracking mechanisms and the development of the damage.

KEY WORDS: composite laminates, transverse cracking, longitudinal cracking, damage, failure criterion

INTRODUCTION

Experimentally, in cross ply laminates subjected to uniaxial static or fatigue tensile loading, it was observed that the damage mechanisms sequences are as follows. The first type of damage observed is generally transverse cracking. After transverse cracking damage, at the crack tip level there is an important interlaminar stress concentration. Generally, this high interlaminar stress provoke on one hand the unsticking of the layers at the interface of the plies with different orientations and/or on the other hand matrix cracking between fibres in the layers parallel with the loading axes. Longitudinal cracking or delamination damage are subsequent development of the transverse cracking damage. For some stacking sequences, the second damage observed is longitudinal damage. This damage is similar to transverse damage, but appears in the 0° layers.

In this article, we propose principally the study and results on the initiation of transverse and longitudinal cracking damage. Generally in the literature, there is no model that studies the problem oflongitudinal cracking damage. The reason is thatthis type ofdamageusually happensnear the endof life the laminate. Another reason is that to study this type of damage, very refineknowledgeof the stressfield distribution in the cracked laminate is required; it is one of the highlights of our approach. With this model, the longitudinal cracks are supposed to be plane and parallel to the 0° layers. The hypotheses of the longitudinal cracks are similar to the hypothesis on the transverses cracks.

In the literature, several approaches have been proposed to investigate the development of cracking damage in cross-ply laminates and several types of criteria have been proposed [1- 5], among them maximum stress based approaches. Other kinds of criteria [6, 7] rely on the energy release rate associated with each type of damage. In this article, an energetic criteria approach is exposed. With this energetic criteria, we have decompose the strain energy of the whole laminate with the purpose to separate component parts related to each component of the stress tensor on one hand and on the other hand, with these results, the different damage mechanisms for each loading mode *(I, II or III)* is proposed. With the damage criterion, the total strain energy release rate is replaced by the part of it which is more directly related to the damage mechanism of interest. For each damage mechanism, the contributions of the loading modes have also been separated.

PROBLEM POSITION

The studied specimen is in fact confined to a $[0_m,90_n]_s$ composite cross-ply laminate as represented in Figure 1. The parameters used to describe the laminate architecture are the λ coefficient ($\lambda = t_0/t_{90}$ where t_0 is the 0° ply thickness and t_{90} is the 90° ply thickness) and the thicknesses of the 0° and 90°plies. The middle plane (xoy plane) of the laminate is a plane of symmetry.

Methodology used with the current approach: to evaluate the strain energy release rates, the laminate is supposed to be previously damaged by transverse and/or longitudinal cracks (Figures 1). The accepted assumptions for the crack geometries in the two layers of the laminate are as follows: All the cracks are supposed to have a rectangular plane geometry and each crack extends over the whole thickness and the whole width in the *90°* damaged ply. Similar assumptions are made for the longitudinal cracks in the *0°* layers. Moreover, the crack distribution is supposed to be uniform along both *x* and *y* directions. With these assumptions, it is sufficient to study the "Unit damaged cell" lies between two consecutive transverse cracks and two consecutive longitudinal cracks. We give a summary of the method used to estimate the stress field distribution in the cracked and delaminated laminate. The analytical model is based on a variational approach relying on the proper choice of a statically admissible stress field. In the damaged laminate, the stress field in the two layers has the following form:

$$\sigma_{ij}^{T(k)} = \sigma_{ij}^{0(k)} + \sigma_{ij}^{P(k)} \qquad (1)$$

For the undamaged laminate loaded in the *x* direction, the layers experience a uniform plane stress state $\sigma_{ij}^{0(k)}$ obtained by the laminate plate theory *(where k is the ply index, k = 0°, 90°).* The orthogonal cracks induce stress perturbations in the *0°* and *90°* layers which are denoted $\sigma_{ij}^{p(k)}$ [8].

THE STRAIN ENERGY RELEASE RATE

The strain energy release rate *G* associated with the initiation and development of the crack damage or delamination damage for a given stress state is defined by the following expression:

$$G = \frac{d}{dA}\widetilde{U}(\sigma, A) \qquad \text{with} \qquad \widetilde{U}_d = N.M.U_{cel} \qquad (2)$$

where $\widetilde{U}_d$ is the strain energy of the whole laminate and Ais the cracked area. Let L_1 denote the laminate length in the x direction, L_2 being its width in the y direction (Figure 1). The strain energy in the damaged unit cell is denoted by U_{cel} . N ($N = L_1/2\bar{a}t_{90}$) is the number of transverse cracks and M ($M = L_2/2\bar{b}t_{90}$) is the number of longitudinal cracks. using dimensionless quantities, $\bar{x} = x/t_{90}$, $\bar{y} = y/t_{90}$, $\bar{z} = z/t_{90}$, $\bar{h} = h/t_{90}$, $\bar{a} = a/t_{90}$, $\bar{b} = b/t_{90}$ and the constraining parameter is $\lambda = t_0/t_{90}$.The transverse crack density is defined by d_t ($d_t = 1/2a$) and the longitudinal crack density is d_l ($d_l = 1/2b$). A_f represente the crack area ($A_f = L_1 L_2 (1/\bar{a} + \lambda/\bar{b})$).

The strain energy release rate associated with transverse cracking is denoted G_{FT} . The strain energy release rate related with longitudinal cracking is denoted G_{FL}.

$$G_{FT} = \frac{d\widetilde{U}d}{dA_f} = \frac{d\widetilde{U}_d}{d\bar{a}} \cdot \frac{d\bar{a}}{dA_f} \qquad G_{FL} = \frac{d\widetilde{U}d}{dA_f} = \frac{d\widetilde{U}_d}{d\bar{b}} \cdot \frac{d\bar{b}}{dA_f} \qquad (3)$$

$$G_{FT} = \frac{1}{2\bar{b}t_{90}^2}\left(U_{cel} - \bar{a}\frac{dU_{cel}}{d\bar{a}}\right) \qquad G_{FL} = \frac{1}{2\bar{a}t_{90}^2\lambda}\left(U_{cel} - \bar{b}\frac{dU_{cel}}{d\bar{b}}\right)$$

THE DECOMPOSITION OF THE STRAIN ENERGY RELEASE RATE

The the strain energy (2) is decomposed to obtain the contributions of all the components product of the stress tensor to the strain energy. The $\widetilde{U}_{ij}$ are related to the products in pairs of the components of the stress tensor is given in the following expression:

$$\widetilde{U}_{ij} = 2.N.M.\left(U_{ij}^{90} + U_{ij}^{0}\right) = \frac{L_1 L_2}{2\bar{a}\bar{b}t_{90}}\left(U_{ij}^{90} + U_{ij}^{0}\right) = \frac{L_1 L_2}{2\bar{a}\bar{b}t_{90}} U_{ij} \qquad (4)$$

Where the *8* parts of the energy $\widetilde{U}_{ij}^{k}$'s are defined by:

$$U_1^{90} = U_{xx}^{90} = \frac{1}{2E_T}\iiint_{V_{90}} \sigma_{xx}^{(90)^2}\, d\bar{x}d\bar{y}d\bar{z} \qquad U_1^{0} = U_{xx}^{0} = \frac{1}{2E_L}\iiint_{V_0} \sigma_{xx}^{(0)^2}\, d\bar{x}d\bar{y}d\bar{z}$$

$$U_2^{90} = U_{yy}^{90} = \frac{1}{2E_L}\iiint_{V_{90}} \sigma_{yy}^{(90)^2}\, d\bar{x}d\bar{y}d\bar{z} \qquad U_2^{0} = U_{yy}^{0} = \frac{1}{2E_T}\iiint_{V_0} \sigma_{yy}^{(0)^2}\, d\bar{x}d\bar{y}d\bar{z}$$

$$U_3^{90} = U_{zz}^{90} = \frac{1}{2E_T}\iiint_{V_{90}} {\sigma_{zz}^{(90)}}^2 \; d\bar{x}d\bar{y}d\bar{z} \qquad U_3^{0} = U_{zz}^{0} = \frac{1}{2E_T}\iiint_{V_{0}} {\sigma_{zz}^{(0)}}^2 \; d\bar{x}d\bar{y}d\bar{z}$$

$$U_4^{90} = U_{xx,zz}^{90} = -\frac{\nu_{LT}}{E_L}\iiint_{V_{90}} \sigma_{xx}^{(90)} \; \sigma_{yy}^{(90)} \; d\bar{x}d\bar{y}d\bar{z} U_4^{0} = U_{xx,zz}^{0} = -\frac{\nu_{LT}}{E_L}\iiint_{V_{0}} \sigma_{xx}^{(0)} \; \sigma_{yy}^{(0)} \; d\bar{x}d\bar{y}d\bar{z} \qquad (5)$$

$$U_5^{90} = U_{yy,zz}^{90} = -\frac{\nu_{LT}}{E_L}\iiint_{V_{90}} \sigma_{yy}^{(90)} \; \sigma_{zz}^{(90)} \; d\bar{x}d\bar{y}d\bar{z} \qquad U_5^{0} = U_{yy,zz}^{0} = -\frac{\nu_{LT}}{E_L}\iiint_{V_{0}} \sigma_{yy}^{(0)} \; \sigma_{zz}^{(0)} \; d\bar{x}d\bar{y}d\bar{z}$$

$$U_6^{90} = U_{xx,zz}^{90} = -\frac{\nu_{TT}}{E_L}\iiint_{V_{90}} \sigma_{xx}^{(90)} \; \sigma_{zz}^{(90)} \; d\bar{x}d\bar{y}d\bar{z} \qquad U_6^{0} = U_{xx,zz}^{0} = -\frac{\nu_{TT}}{E_L}\iiint_{V_{0}} \sigma_{xx}^{(0)} \; \sigma_{zz}^{(0)} \; d\bar{x}d\bar{y}d\bar{z}$$

$$U_7^{90} = U_{xz}^{90} = \frac{1}{2G_{TT'}}\iiint_{V_{90}} {\sigma_{xz}^{(90)}}^2 \; d\bar{x}d\bar{y}d\bar{z} \qquad U_7^{0} = U_{xz}^{0} = \frac{1}{2G_{LT}}\iiint_{V_{0}} {\sigma_{xz}^{(0)}}^2 \; d\bar{x}d\bar{y}d\bar{z}$$

$$U_8^{90} = U_{yz}^{90} = \frac{1}{2G_{LT}}\iiint_{V_{90}} {\sigma_{yz}^{(90)}}^2 \; d\bar{x}d\bar{y}d\bar{z} \qquad U_8^{0} = U_{yz}^{0} = \frac{1}{2G_{TT'}}\iiint_{V_{0}} {\sigma_{yz}^{(0)}}^2 \; d\bar{x}d\bar{y}d\bar{z}$$

The contribution of each selected component pair to the strain energy release rate is such that:

$$G_{ij}^{k} = \frac{d\bar{U}_{ij}^{k}}{dA} \qquad \text{where } i = x, y, z \text{ and } k = 0°, 90° \qquad (6)$$

The main objective of this paper is to propose appropriate approximations of the strain energy release rates for each type of damage (transverse cracking and longitudinal cracking) for the three damage modes (mode *I(opening mode)*, mode *II(sliding mode)* and mode *III (tearing mode))* [6, 9]. We computed all the parts of the strain energy release rate attributable to each stress component. Thus, the total strain energy release rate is replaced by only one of its component in (5) for describing the evolution of each damage mechanism and loading mode.

The estimation of the of the strain energy release rate, associated to the loading mode *(mode I, II* and *III)* [10] for the transverse crack damage *(FT)* and longitudinal crack damage *(FL)* are obtained with the following expressions:

$$G_I^{FT} = \frac{d\bar{U}_{xx}}{d\bar{a}} \cdot \frac{d\bar{a}}{dA_f} \qquad G_{II}^{FT} = \frac{d\bar{U}_{xz}}{d\bar{a}} \cdot \frac{d\bar{a}}{dA_f}$$

$$G_I^{FL} = \frac{d\bar{U}_{yy}}{d\bar{b}} \cdot \frac{d\bar{b}}{dA_f} \qquad G_{II}^{FL} = \frac{d\bar{U}_{yz}}{d\bar{b}} \cdot \frac{d\bar{b}}{dA_f} \qquad (7)$$

We estimated all the components of the strain energy release rates associated to each type of damage and for each fracture mode (7). To predict the evolution of the different damage mechanisms, a damage criterion is proposed. Each estimated quantity was compared with the associated critical value of the strain energy release rate.

$$G = G_c \qquad (8)$$

For instance, as regards the transverse cracking damage, we replaced the total strain energy release rate by the approximate quantity relative to the opening mode (*mode I*) and the sliding mode (*mode II*). The two computed values are compared with associated critical values G_{crf}. For example in [11, 12] G_{crf} values for a graphite epoxy system are given. In regards to the initiation and development of other types of damage, longitudinal cracking, the transverse crack density is fixed. For this crack density value, the partial strain energy release rates (7) are estimated. All the partial values are compared with critical values G_{crf} in the criterion (8). The strain energy release rates pertaining to each damage mechanism and mode governs the variation of this type of damage, the other types of damage being fixed.

Results

The energetical criterion proposed is elastic linear fracture based approach. With this criterion, a decomposition of the strain energy release rate associated with each type of damage is used. A by-product of this study is the assessment of the architecture and material system influence on the initiation of transverse cracking and longitudinal cracking. The parameters used in the study are the constraining parameter, the thickness of the two *0°* and *90°* layers and the material constituent system. The constraining parameter is λ $(\lambda = m/n)$, where n is the number of plies with *90°* orientation and m is the number of plies with *0°* orientation. In all the proposed results, the numerical simulations are carried out for a prescribed uni-axial loading of *150 MPa*. The composite material systems studied is a graphite/epoxy *T300-934* system (Table 2). In some numerical simulations, the partial parts of the strain energy release rates, associated with the initiation of transverse cracking and longitudinal cracking are normalized by the critical strain energy release rate. The critical value of the strain energy release rate associated with transverse and longitudinal cracking initiation is denoted G_{crf} .

In Figure 2the data pertain of a graphite/epoxy laminate and the results concern the decomposition of the strain energy release rate versus the constraining parameter lambda. We can observe the evolution of the decomposition of by-product in the 90° layer of the laminate versus the constraining parameter. The part due to the stress in the loading direction of the laminate is preponderant.

In Figure 3, the results of the numerical simulations confirm two main points: the proposed approach agrees with experimental data for the initiation of transverse cracking as the first damage mode. It also predicts the readiness to initiate the three types of damage in the case of a *8* ply laminate containing a thick *90°* layer.

The result in Figure 3 show that the part attributed to the 90° the most important part of the strain energy release rate.

Figure 4 only shows numerical simulations of the strain energy release rate evolutions associated longitudinal cracking. The decomposition proposed is similar to the results exposed in the Figure 2, but for the estimation of the partial part of the strain energy associated to longitudinal damage. The result in Figure 5 show that the part attributed to the 0° on the strain energy release rate.

We can also observe that the strain energy release rates, G_{FT} and G_{FL} have similar variation laws. All the strain energy release rates are decreasing functions of the constraining parameter □. For instance, in a *8* ply laminate, when the value of the constraining parameter □□ is increased, the thickness of the *0°* plies becomes greater. In this case, the fibers in the *0°* plies carry most of the tensile loading and the initiation of the three different damage modes is delayed.

The main objective of this paper is to present results of an energetic criterion for estimating the initiation of several damage mechanisms. The proposed results concern numerical simulations pertaining to various approximations. These approximations concern the decomposition of the strain energy release associated to the different damage mechanisms and fracture modes. In Table 1, the proposed decomposition of the strain energy release rate, associated with the initiation of transverse cracks and longitudinal cracks, is also related to the different loading modes (mode *I(opening mode)*, mode *II(sliding mode)* and mode *III (tearing mode))* which control the three types of damage. With thisdecomposition (Figure 2 and 4), we can observe the part ofeach component of thestress tensorand their influence ontheevolution of the damage. In Figure6, the results of the numerical simulations show that transverse cracking and longitudinal cracking are initiated in opening mode *(Mode I).* Numerous numerical simulations, with other materials and laminate architectures, give similar evolutions of the cracking damage. The decomposition of the strain energy release rate associated to the initiation of the transverse and longitudinal cracks highlights the different modes that drive the two types of damage mode. Thus, from the proposed numerical simulations, we can conclude that transverse cracking is mostly controlled by the normal stress $\sigma_{xx}^{(90)}$, whereas longitudinal cracking is mostly controlled by the normal stress $\sigma_{yy}^{(90)}$. The effect of the sliding mode (mode *II*) is note significant. With this approach, the part of the tearing mode (mode *III*) is not observed.

Figure 1 :*Laminate damaged by transverse cracks and longitudinal cracks*.

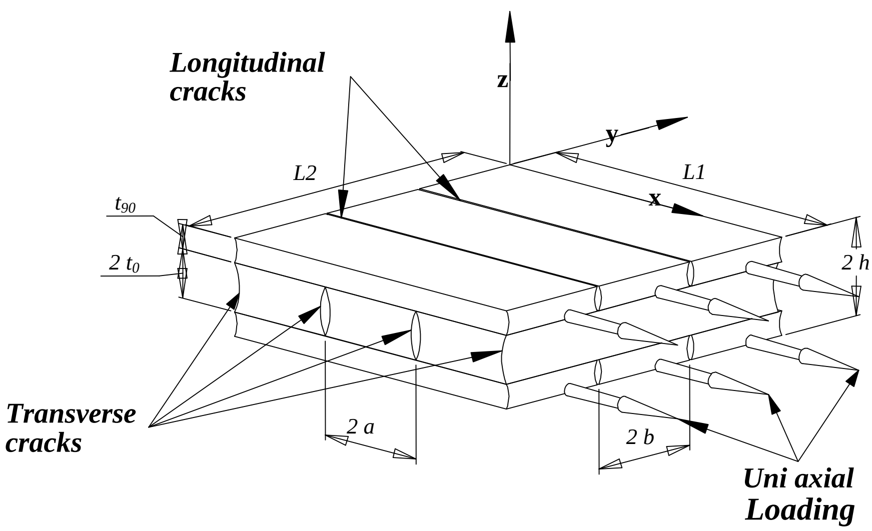

Figure 2: *Variation of the partial part of the strain energy release rate for transverse damage versus the constraining parameter* .

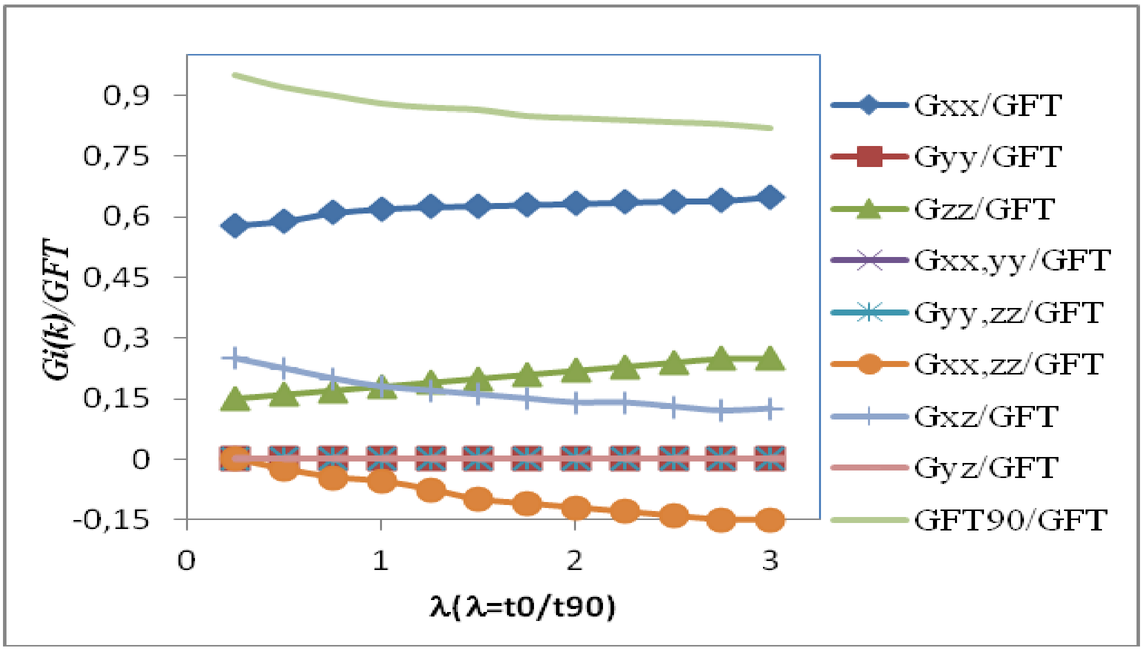

Figure 3: *:Variation of the layer contribution to the strain energy release rate (G_FT= G_FT^0+ G_FT^90), for transverse damage with the constraining parameter* .

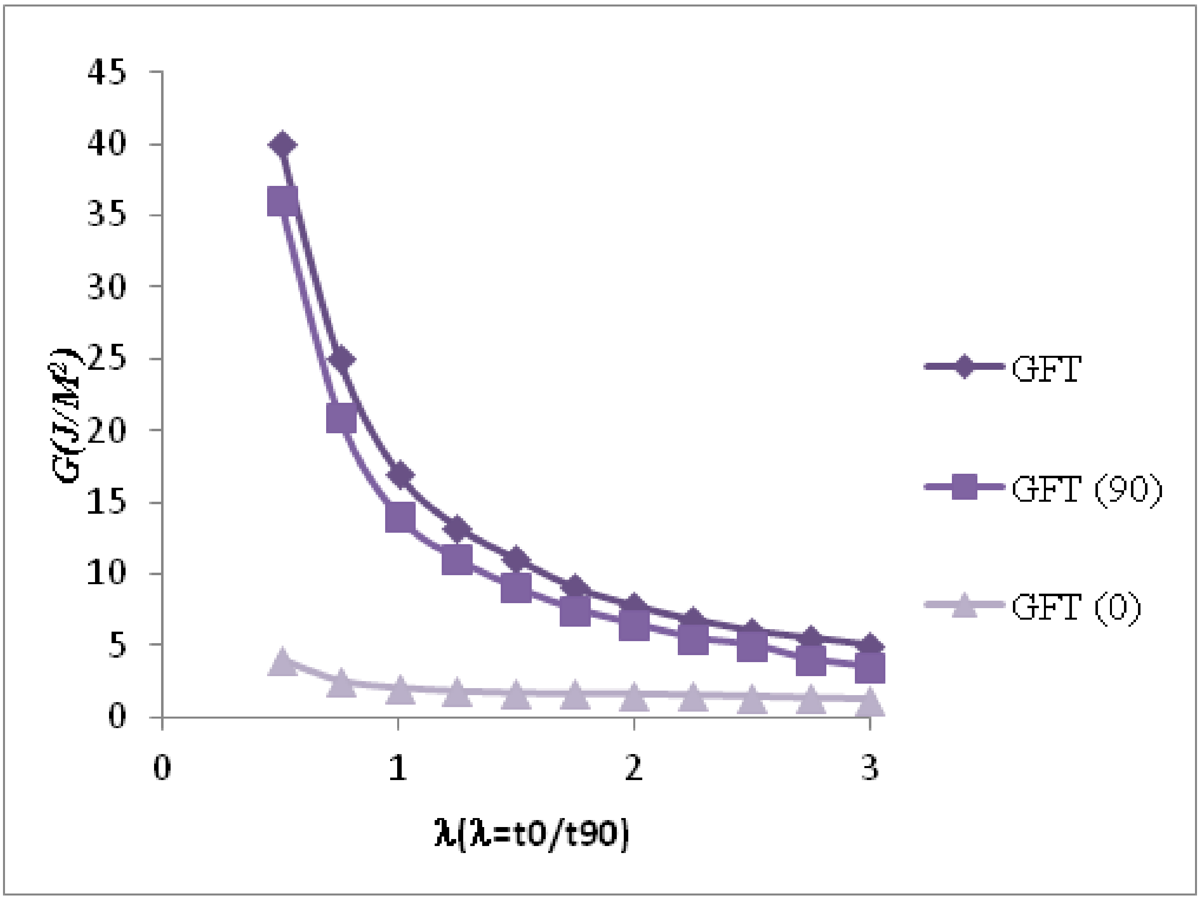

Figure 4: *Variation of the partial part of the strain energy release rate for longitudinal damage with the constraining parameter .*

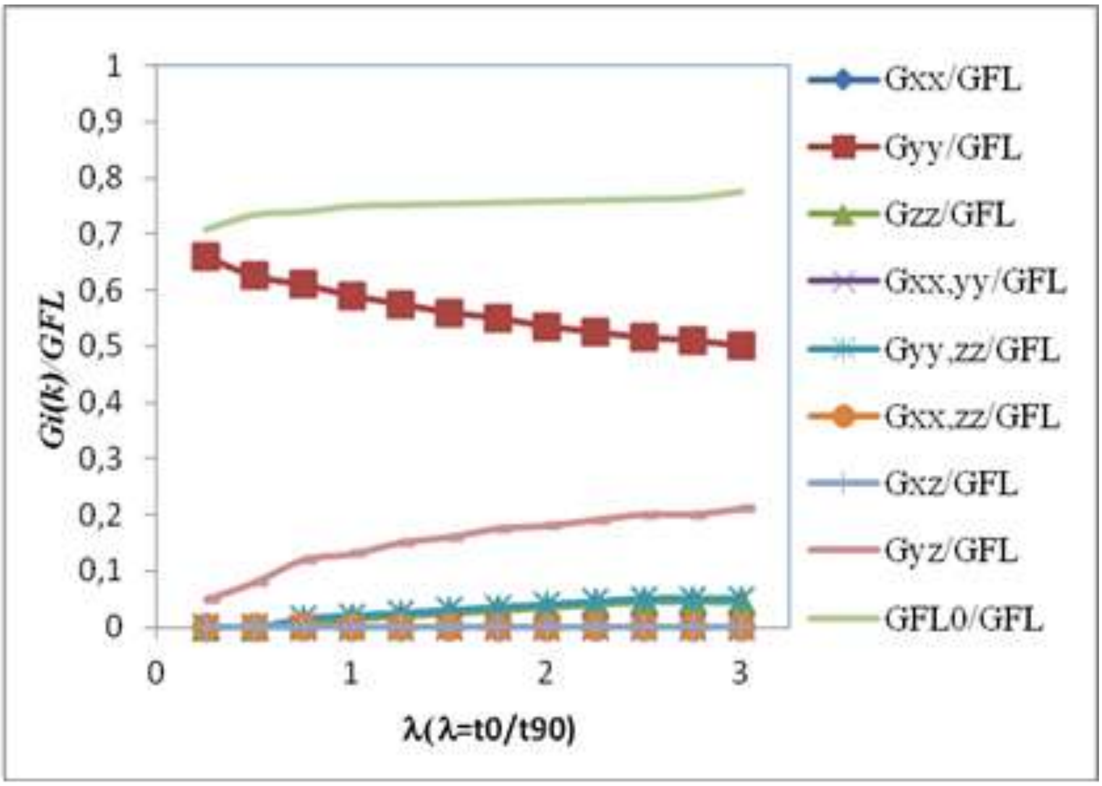

Figure 5: *Variation of the layer contribution to the strain energy release rate (G_FL= G_FL^0+ G_FL^90), for transverse damage with the constraining parameter .*

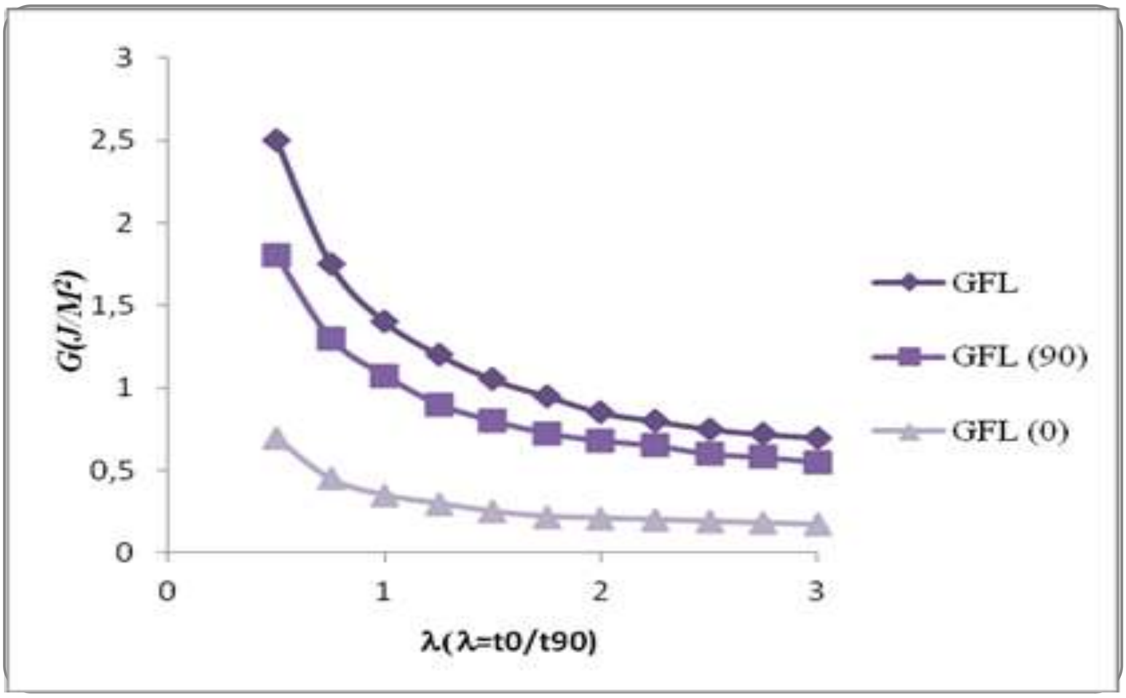

Figure 6: *The initiation of transverse cracks in a 8-ply laminate with constraining parameter ⍰ ⍰ ⍰ is controlled by the breaking modes G_I⁄G_FT , G_II⁄G_FT , G_I⁄G_FL , G_II⁄G_FL*

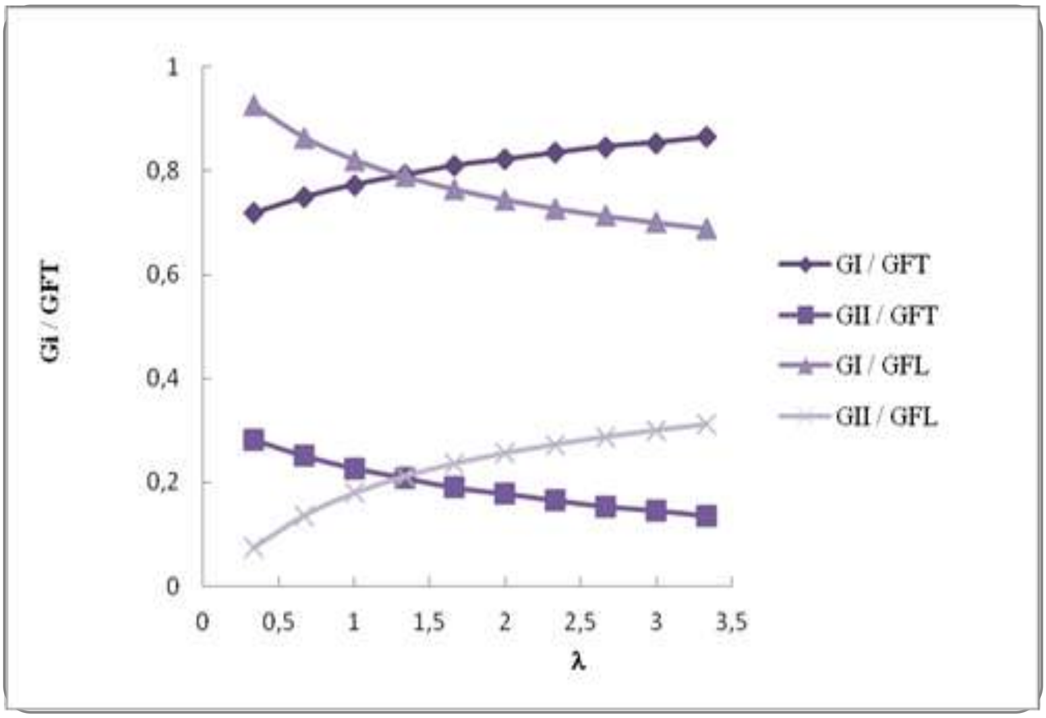

Table 1: *Stress components with different mechanisms and loading modes.*

	Transverse cracking	Longitudinal cracking
Mode I	$\sigma_{xx}^{(90)}$	$\sigma_{yy}^{(0)}$
Mode II	$\sigma_{xz}^{(90)}$	$\sigma_{yz}^{(0)}$

Mode III	$\sigma_{xy}^{(90)}$	$\sigma_{xy}^{(0)}$

Table 2: *Mechanical properties and ply thickness for T300/934 graphite epoxy system*

	Carbone/ Époxy
E_{LT} (GPa)	140
$E_{TT'}$(GPa)	10
G_{LT} (GPa)	5.7
$G_{TT'}$(GPa)	3.6
ν_{LT}	0.31
$\nu_{TT'}$	0.58
Ply thickness (mm)	0.125
G_{crf} (J/m^2)	228

CONCLUSION

The objective of this paper is to give an approximate expression for the strain energy release rate associated with each cracking mechanism and mode. The computation of the "partial" strain energy release rates is achieved according to Table 1. The related decomposition was numerically assessed to understand and predict the most probable damage mechanism occurrence. In the type of cross-ply laminates investigated, several damage mechanisms appear, thesemechanisms can befollowed up withthis kind of approach.

Using some simplifying hypotheses, the relevant fracture mechanism and mode is predicted through a strain energy release rate criterion. From the proposed approach, several results can be obtained: it is confirmed that transverse matrix cracking is usually the first damage mechanism observed. In all the cases investigated, transverse cracking and longitudinal cracking are initiated in opening mode *(mode I)*, sliding mode (mode *II*) is note significant and tearing mode (mode *III*) unobserved.

REFERENCES

[1] Akshantala N-V, Talreja R. A micro mechanics model for predicting fatigue life of composite laminate. Materials Science and Engineering. 2000, A285: 303-313.

[2] Pereira AB, de Morais AB, Marques AT, de Castro PT. Mode II interlaminar fracture of carbon/epoxy cross-ply multidirectional laminates. Composites Science and Technology. 2002, 62: 679-686.

[3] Dakshina Moorthy CM, Reddy JN. Recovery of interlaminar stresses and strain energy release rate in composite laminates. Finite Elements in Analysis and Design. 1999, 33: 1-27.

[4] Hinton MJ, Kaddour AS, Soden PD. Evaluation of failure prediction in composite laminates. Composites Science and Technology. 2004, 64 (3-4): 321-327.

[5] Xia Z, Chen Y, Ellyin F. A meso/micro mechanical model for damage progression in glass fiber/epoxy cross-ply laminates by finite-element analysis. Composites Science and Technology. 2000, 60: 1171-1179.

[6] Rebiиre J.-L., Gamby D., A criterion for modelling initiation and propagation of matrix cracking and delamination in cross-ply laminates, Composites Science and Technology. 2004, 64: 2239-2250.

[7] Rebiиre J-L. Modйlisation du champ des contraintes crйй par des fissures de fatigue dans un composite stratifiй carbone/polymиre. Thиse de doctorat (in French), Universitй de Poitiers, 1992.

[8] Rebиre J-L, Maвtallah M-N, Gamby D. Initiation and growth of transverse and longitudinal cracks in composite cross-ply laminates. Composite Structures. 2001, 53: 173-187.

[9] Lemaotre J, Chaboche J-L. Mйcanique des matйriaux solides. Dunod 2nd Edition; 1988.

[10] Mayes JS, Hansen AC. A comparison of multicontinuum theory based failure simulation with experimental results. Composites Science and Technology. 2004, 64 (3-4): 517-527.

[11] Wang ASD, Kishore NM, Li CA. Crack development in graphite-epoxy cross-ply laminates under uniaxial tension. Composites Science and Technology. 1985, 24: 1-31.

[12] Wang ASD, Crossman FW. Initiation and growth of transverse cracks and edge delamination in composite laminates. Part I : An energy method. Journal of Composite Materials. 1980, 14 Sup.: 71-87.